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Screening of cosmological constant in non-local gravity

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We discuss a possible mechanism to screen a cosmological constant in non-local gravity. We find that in a simple model of of non-local gravity with the Lagrangian of the form, $R + f(\Box^{-1}R) - 2\Lambda$ where $f(X)$ is a quadratic function of X, there is a flat spacetime solution despite the presence of the cosmological constant Λ. Unfortunately, however, we also find that this solution contains a ghost in general. Then we discuss the condition to avoid a ghost and find that one can avoid it only for a finite range of 'time'. Nevertheless our result suggests the possibility of solving the cosmological constant problem in the context of non-local gravity.

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I. INTRODUCTION

The study of string/M-theory or conventional quantum gravity, for instance, using the effective action approach often leads to the appearance of non-local gravitational terms. These terms are not easy to deal with, nevertheless, they maybe important in the applications for early-time and/or late-time universe. In particular, they maybe responsible for early/late-time acceleration. Recently, there has been increasing interest in various aspects of non-local gravity[1–4, 6, 7]. Various different theories of non-local gravity have been proposed in these works and their accelerating FLRW cosmology was investigated. It was found that in the same way as other modified gravity theories non-local gravity may (at least, qualitatively) lead to a unified description of the early-time inflation and the present dark energy era (for review of such an approach, see [8]).

The present paper is devoted to a related aspect of non-local gravity: the possible solution to the cosmological constant problem. Indeed, it was suggested sometime ago that the cosmological constant problem may be solved by non-local gravity [5]. Here we give an explicit example for such a proposal. In the next section the model of non-local gravity is considered and its scalar-tensor presentation is given. Its ghost-free condition is analyzed in section III. A simple example which is ghost-free and which provides the solution to the cosmological constant problem is given in section IV. Summary is given in Discussion section. In Appendix we propose a new model of non-local gravity with a Lagrange constraint multiplier.

II. NON-LOCAL GRAVITY

The starting action of the non-local gravity is given by

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left\{R\left(1 + f(\Box^{-1}R)\right) - 2\Lambda\right\} + \mathcal{L}_{\rm matter}\left(Q;g\right)\right\} . \tag{1}$$

Here f is some function, $\Box$ is the d'Alembertian for scalar field, Λ is a cosmological constant and Q stands for the matter fields. The above action can be rewritten by introducing two scalar fields ψ and ξ in the following form,

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\psi)\right)-\xi\left(\Box\psi-R\right)-2\Lambda\right\}+\mathcal{L}_{\rm matter}\right] \\
&= \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\psi)+\xi\right)+g^{\mu\nu}\partial_\mu\xi\partial_\nu\psi-2\Lambda\right\}+\mathcal{L}_{\rm matter}\right]. \qquad (2)
\end{aligned}
$$

By varying the above with respect to ξ, we obtain $\Box\psi = R$ or $\psi = \Box^{-1}R$. Substituting the above equation into (2), one re-obtains (1).

By varying (2) with respect to the metric $g_{\mu\nu}$ gives

$$
\begin{aligned}
0 =& \frac{1}{2}g_{\mu\nu}\left\{R\left(1+f(\psi)+\xi\right)+g^{\alpha\beta}\partial_\alpha\xi\partial_\beta\psi-2\Lambda\right\}-R_{\mu\nu}\left(1+f(\psi)+\xi\right) \\
&-\frac{1}{2}\left(\partial_\mu\xi\partial_\nu\psi+\partial_\mu\psi\partial_\nu\xi\right)-\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)\left(f(\psi)+\xi\right)+\kappa^2T_{\mu\nu}\,. \qquad (3)
\end{aligned}
$$

On the other hand, the variation with respect to ψ gives

$$
0 = \Box\xi - f'(\psi)R\,. \qquad (4)
$$

Now we assume the FLRW metric

$$
ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j\,, \qquad (5)
$$

and the scalar fields ψ and ξ only depend on time. Then Eq. (3) takes the following form,

$$
0 = -3H^2\left(1+f(\psi)+\xi\right)-\frac{1}{2}\dot\xi\dot\psi-3H\left(f'(\psi)\dot\psi+\dot\xi\right)+\Lambda+\kappa^2\rho\,, \qquad (6)
$$

$$
0 = \left(2\dot H+3H^2\right)\left(1+f(\psi)+\xi\right)-\frac{1}{2}\dot\xi\dot\psi+\left(\frac{d^2}{dt^2}+2H\frac{d}{dt}\right)\left(f(\psi)+\xi\right)-\Lambda+\kappa^2P\,, \qquad (7)
$$

where $H = \dot a/a$, and $\rho = -T^0{}_0$ and $P = T^i{}_i/3$ are the energy density and pressure, respectively, of the matter fields. On the other hand, the scalar equations are

$$
0 = \ddot\psi + 3H\dot\psi + 6\dot H + 12H^2\,, \qquad (8)
$$

$$
0 = \ddot\xi + 3H\dot\xi + \left(6\dot H + 12H^2\right)f'(\psi)\,. \qquad (9)
$$

III. GHOST-FREE CONDITION

Let $\tilde g_{\mu\nu}$ be the original metric discussed in the previous section. We make a conformal transformation to the Einstein frame,

$$
\tilde g_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \quad \tilde R = \frac{1}{\Omega^2}\left[R-6\left(\Box\ln\Omega+g^{\mu\nu}\nabla_\mu\ln\Omega\nabla_\nu\ln\Omega\right)\right]\,, \qquad (10)
$$

with

$$
\Omega^2 = \frac{1}{1+f(\psi)+\xi}\,. \qquad (11)
$$

This gives

$$
\begin{aligned}
S =& \int d^4x\sqrt{-g}\Bigg\{\frac{1}{2\kappa^2}\left[R-6\left(\Box\ln\Omega+g^{\mu\nu}\nabla_\mu\ln\Omega\nabla_\nu\ln\Omega\right)+\Omega^2g^{\mu\nu}\nabla_\mu\xi\nabla_\nu\psi-2\Omega^4\Lambda\right] \\
&+\Omega^4\mathcal{L}_{\rm matter}(Q;\Omega^2 g)\Bigg\}\,. \qquad (12)
\end{aligned}
$$

The $\Box \ln \Omega$ term may be discarded because it is a total divergence. Hence we obtain

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R - 6g^{\mu\nu}\nabla_\mu \ln\Omega \nabla_\nu \ln\Omega + \Omega^2 g^{\mu\nu}\nabla_\mu \xi \nabla_\nu \psi - 2\Omega^4\Lambda\right] + \Omega^4 \mathcal{L}_{\rm matter}(Q;\Omega^2 g)\right\} \\ &= \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R - 6g^{\mu\nu}\nabla_\mu \phi \nabla_\nu \phi + e^{2\phi} g^{\mu\nu}\nabla_\mu \xi \nabla_\nu \psi - 2e^{4\phi}\Lambda\right] + e^{4\phi} \mathcal{L}_{\rm matter}(Q;e^{2\phi} g)\right\} . \end{aligned} \tag{13}$$

where

$$\phi \equiv \ln\Omega = -\frac{1}{2}\ln(1+f(\psi)+\xi)\,. \tag{14}$$

Here we note the condition for the gravity to have the normal sign,

$$1 + f(\psi) + \xi > 0\,. \tag{15}$$

Instead of ψ and ξ, one may regard ϕ and ψ to be independent fields. Then inserting

$$\xi = e^{-2\phi} - (1 + f(\psi)) \tag{16}$$

into (13), we finally arrive at

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R - 6\nabla^\mu\phi\nabla_\mu\phi - 2\nabla^\mu\phi\nabla_\mu\psi - e^{2\phi}f'(\psi)\nabla^\mu\psi\nabla_\mu\psi - 2e^{4\phi}\Lambda\right] + e^{4\phi}\mathcal{L}_{\rm matter}(Q;e^{2\phi}g)\right\} . \tag{17}$$

In order to avoid a ghost, the determinant of the kinetic term must be positive. This means

$$\det\begin{vmatrix} 6 & 1 \\ 1 & e^{2\phi}f'(\psi) \end{vmatrix} = 6e^{2\phi}f'(\psi) - 1 > 0\,. \tag{18}$$

We assume this condition is satisfied. In particular, $f'(\psi) > 0$ is a necessary condition.

To summarize, in terms of the original scalar fields, the condition for the theory to be healthy is

$$f'(\psi) > \frac{1+f(\psi)+\xi}{6} > 0\,. \tag{19}$$

IV. SIMPLE MODEL

We consider a simple model for $f(\psi)$:

$$f(\psi) = f_0\psi + f_1\psi^2\,, \tag{20}$$

and look for the flat spacetime solution $H = 0$. For simplicity we assume the matter to be absent, $\rho = P = 0$. Then from the scalar equations (8) and (9), the solutions for $\psi(t)$ and $\xi(t)$ can be obtained as

$$\psi(t) = \psi_0 + \psi_1 t, \tag{21}$$

$$\xi(t) = \xi_0 + \xi_1 t, \tag{22}$$

where ψ_0, ψ_1, ξ_0 and ξ_1 are integral constants. Without loss of generality we may put $\psi_0 = \xi_0 = 0$. Also we may assume $\psi_1 > 0$ because of the time reversal invariance. Inserting these to Einstein equations (6) and (7), we find

$$\xi_1 = 2\Lambda\sqrt{\frac{f_1}{\Lambda}}, \tag{23}$$

$$\psi_1 = \sqrt{\frac{\Lambda}{f_1}}\,. \tag{24}$$

Here we note that f_1 must have the same sign as that of Λ.

Thus we have found that there is a flat spacetime solution in this non-local gravity model even under the presence of a cosmological constant. In other words, the non-local gravitational effect has successfully shielded the effect of a cosmological constant.

Now we discuss the condition to avoid the appearance of a ghost in the above solution. Let us recapitulate the condition (19),

$$f'(\psi) > \frac{1+f(\psi)+\xi}{6} > 0\,. \tag{25}$$

In our model, using Eqs. (23) and (24), the above condition gives

$$G(t) \equiv t^2\Lambda + t\sqrt{\frac{\Lambda}{f_1}}(f_0 - 10f_1) + 1 - 6f_0 < 0, \tag{26}$$

$$K(t) \equiv t^2\Lambda + t\sqrt{\frac{\Lambda}{f_1}}(f_0 + 2f_1) + 1 > 0. \tag{27}$$

Below we discuss these conditions in the cases of positive and negative cosmological constants separately.

A. $\Lambda > 0$ case

First, we discuss the case when $\Lambda > 0$. We assume $f_1 > 0$ for consistency. Then the necessary condition to satisfy (26) is

$$\Delta_G \equiv \Lambda\frac{(f_0+2f_1)^2 - 4f_1 + 96f_1^2}{f_1} > 0, \tag{28}$$

which directly leads to the condition,

$$f_0^2 + 4f_1(f_0 - 1) > -100f_1^2. \tag{29}$$

Let t_1 and t_2 be two solutions for $G(t) = 0$, we have

$$t_{1,2} = \frac{1}{\Lambda}\sqrt{\frac{\Lambda}{f_1}}\left\{-\frac{f_0}{2} + 5f_1 \pm \left[\frac{f_0^2}{4} + f_1\left(25f_1 + f_0 - 1\right)\right]^{\frac{1}{2}}\right\}, \tag{30}$$

where $t_1 < t_2$ is assumed so that the range of t is between t_1 and t_2, i.e. $t \in (t_1, t_2)$.

The sufficient condition to satisfy (27) is

$$\Delta_K \equiv \Lambda\frac{(f_0+2f_1)^2 - 4f_1}{f_1} < 0\,, \tag{31}$$

that is,

$$f_0^2 + 4f_1(f_0 - 1) < -4f_1^2. \tag{32}$$

Thus, combining Eqs. (29) and (32) together, we conclude that for a positive Λ the sufficient condition to prevent a ghost is

$$-25f_1^2 < \frac{f_0^2}{4} + f_1f_0 - f_1 < -f_1^2\,, \tag{33}$$

or

$$\frac{1}{f_1} - 25 < \frac{f_0^2}{4f_1^2} + \frac{f_0}{f_1} < \frac{1}{f_1} - 1\,. \tag{34}$$

The range of time Δt_+ is

$$\Delta t_+ = 2\sqrt{\frac{f_1}{\Lambda}}\left(\frac{f_0^2}{4f_1^2} + \frac{f_0}{f_1} + 25 - \frac{1}{f_1}\right)^{\frac{1}{2}}. \tag{35}$$

B. $\Lambda < 0$ case

Now we turn to discuss the situation when $\Lambda < 0$. In this case we assume $f_1 < 0$ for consistency. The main difference from the case of a positive Λ is that the condition (28) is replaced by $\Delta_K > 0$,

$$\Delta_K = \Lambda \left[\frac{(f_0 + 2f_1)^2}{f_1} - 4 \right] > 0 \,, \tag{36}$$

which leads to

$$f_0^2 + 4f_1(f_0 - 1) > -4f_1^2. \tag{37}$$

Since $\Delta_G = \Delta_K + 96 f_1 \Lambda > \Delta_K$, Δ_G is always positive if $\Delta_K > 0$. Thus $G(t) = 0$ has two real solutions if $K(t) = 0$ has two real solutions.

Let t_3 and t_4 be the solutions for $K(t) = 0$, then we have

$$t_{3,4} = \frac{1}{\Lambda} \sqrt{\frac{\Lambda}{f_1}} \left\{ -\frac{f_0}{2} - f_1 \pm \left[\frac{f_0^2}{4} + f_1 \left(f_1 + f_0 - 1 \right) \right]^{\frac{1}{2}} \right\} , \tag{38}$$

where we assume $t_3 < t_4$. While the solutions for $G(t) = 0$ have been already given by $t_{1,2}$ in (30).

Thus, if we have $t_4 < t_1$ or $t_2 < t_3$ for a non-zero range of t, the range of t is given by $t \in (t_3, t_4)$. Both of these requirements yield the same condition,

$$f_0^2 + 4f_1(f_0 - 1) < 0. \tag{39}$$

So we conclude that for $\Lambda < 0$, the sufficient condition to avoid a ghost is

$$-4f_1^2 < f_0^2 + 4f_1(f_0 - 1) < 0, \tag{40}$$

or

$$\frac{1}{f_1} - 1 < \frac{f_0^2}{4f_1^2} + \frac{f_0}{f_1} < \frac{1}{f_1} \,. \tag{41}$$

The range of time Δt_- is

$$\Delta t_- = 2 \sqrt{\frac{f_1}{\Lambda}} \left(\frac{f_0^2}{4f_1^2} + \frac{f_0}{f_1} + 1 - \frac{1}{f_1} \right)^{\frac{1}{2}} . \tag{42}$$

In both cases we conclude that the typical range of time for our model to be healthy is of order $\sqrt{f_1/\Lambda}$ provided that f_0 and f_1 are of the same order.

Note that it is known that the theory under consideration admits also de Sitter type solution[1] with or without the initial large cosmological constant. The proposed scenario for the effective screening of cosmological constant works also in this case. It will be considered elsewhere.

V. DISCUSSION

Let us consider a specific example for the values of the model parameters. Let us assume that the "bare" cosmological constant Λ in the action (1) is very large, say of the order of Planck scale, or of GUT scale $|\Lambda|\kappa^2 \sim 1 - 10^{-12}$. Compared to these values, the cosmological constant that could explain the accelerated expansion of the present universe is negligibly small, $\Lambda_0 \kappa^2 \sim (10^{-3}\,\mathrm{eV})^4/(10^{18}\,\mathrm{Gev})^4 \sim 10^{-120}$. So we may regard the present universe as a flat spacetime at leading order approximation. In this case, however, we would need our universe to be stable for a sufficiently long time, or the ghost-free period should be large enough, $\Delta t \gtrsim (\Lambda_0)^{-1/2} \sim 10^{60}\kappa$.

For definiteness let us consider the case of $\Lambda > 0$. If we rewrite Eq. (34) as

$$\frac{1}{f_1} - 24 < \left(\frac{f_0}{2f_1} + 1 \right)^2 < \frac{1}{f_1} \,, \tag{43}$$

we immediately see that this condition is satisfied if $f_1 \gg 1$ and $f_0 = -2f_1$. That is,

$$f(\psi) = f_1 \left[(\psi - 1)^2 - 1\right] . \tag{44}$$

Then from Eqs. (20)-(24), the time interval Δt_+ becomes

$$\Delta t_+ = 2\sqrt{\frac{f_1}{\Lambda}} \left(24 - \frac{1}{f_1}\right)^{\frac{1}{2}} \approx 10\sqrt{\frac{f_1}{\Lambda}} . \tag{45}$$

Thus to have $\Delta t \gtrsim (\Lambda_0)^{-1/2} \sim 10^{60}\kappa$ we would need f_1 to be extremely large. In this sense, unfortunately our model cannot be regarded as a complete solution to the cosmological constant problem, since we need an unnaturally large number in the model. Nevertheless, it gives us a hope that for a more sophisticated model of non-local gravity, it may be possible to have a solution that screens the bare cosmological constant without encountering the problem of ghosts.

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Appendix A: Non-local gravity with Lagrange constraint multiplier

The idea proposed in [9] for quantum gravity is to modify the ultraviolet behavior of the graviton propagator in Lorentz non-invariant way as $1/|\boldsymbol{k}|^{2z}$, where $\boldsymbol{k}$ is the spatial momenta and z could be 2, 3 or larger integers. They are defined by the scaling properties of space-time coordinates $(\boldsymbol{x}, t)$ as follows,

$$\boldsymbol{x} \to b\boldsymbol{x} , \quad t \to b^z t . \tag{A1}$$

When $z = 3$, the theory seems to be (power counting) UV renormalizable. Then in order to realize the Lorentz non-invariance, one introduces the terms breaking the Lorentz invariance explicitly (or more precisely, breaking full diffeomorphism invariance) by treating the temporal coordinate and the spatial coordinates in a different way. Such model has the diffeomorphism invariance with respect only to the time coordinate t and spatial coordinates $\boldsymbol{x}$:

$$\delta x^i = \zeta^i(t, \boldsymbol{x}) , \quad \delta t = f(t) . \tag{A2}$$

Here $\zeta^i(t, \boldsymbol{x})$ and $f(t)$ are arbitrary functions.

In [11], Hořava-like gravity model with full diffeomorphism invariance has been proposed. When we consider the perturbations from the flat background, which has Lorentz invariance, the Lorentz invariance of the propagator is dynamically broken by the non-standard coupling with a perfect fluid. The obtained propagator behaves as $1/\boldsymbol{k}^{2z}$ with $z = 2, 3, \cdots$ in the ultraviolet region and the model could be perturbatively power counting (super-)renormalizable if $z \geq 3$. In [12] it was proposed the model of covariant and power-counting renormalizable field theory of the gravity. The essential element of the construction is Lagrange multiplier constraint proposed in [10].

Let us show here that our non-local gravity maybe generalized for the case of introduction of such Lagrange multiplier constraint in an easy way.

Under the constraint,

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0 = 0 , \tag{A3}$$

we now define

$$\begin{aligned} R^{(2n+1)} &\equiv R - 2\kappa^2\alpha \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^n \left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right) \right\}^2 , \\ R^{(2n+2)} &\equiv R - 2\kappa^2\alpha \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^n \left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right) \right\} \\ &\quad \times \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^{n+1} \left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right) \right\} , \\ \Box^{(n)} &\equiv \Box + \beta \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^n . \end{aligned} \tag{A4}$$

In the same way as Eq. (1), one may define the non-local action,

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left\{R^{(m)}\left(1+f\left(\left(\Box^n\right)^{-1}R^{(m)}\right)\right)-2\Lambda\right\}-\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+U_0\right)+\mathcal{L}_{\rm matter}\right\}. \tag{A5}$$

and rewrite the action (A5) in a local way by introducing two scalar fields η and ξ:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R^{(m)}\left(1+f(\eta)\right)+\xi\left(\Box^{(n)}\eta-R^{(m)}\right)-2\Lambda\right\}-\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+U_0\right)+\mathcal{L}_{\rm matter}\right]. \tag{A6}$$

In (A5) and (A6), n can be either even or odd integer. This (power-counting) non-local gravity also admits a flat space solution in the presence of cosmological constant. Such a solution has similar properties as in the model of IV section and may also describe the screening of cosmological constant.

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